

**PRAGATI ENGINEERING COLLEGE: SURAMPALEM
(AUTONOMOUS)**

I B.Tech II Semester Regular Examinations, June-2024

**DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS
(Common to all branches)**

Time: 3 hours

Max. Marks: 70

Note:

- i. Question No. 1 shall contain 10 compulsory short answer questions (2 questions from each unit) for a total of 20 marks such that each question carries 2 marks.
- ii. In each of the questions from 2 to the last question, there shall be either/or type questions of 10 marks each.

.Q. No.	Questions	BTL	CO	Marks
1.	a) Define an exact equation. What is the general solution of an exact equation?	K1	CO1	2M
	b) State Newton's law of cooling and give its mathematical form.	K1	CO1	2M
	c) Find the general solution of $(D^3 - a^3)y = 0$.	K2	CO2	2M
	d) Define the Wronskian.	K1	CO2	2M
	e) Form a partial differential equation from $z = ax^2 + by^2$.	K1	CO3	2M
	f) Form a partial differential equation from $z = f(ax + by)$.	K2	CO3	2M
	g) Define the gradient of a scalar point function $f(x, y, z)$. What is the gradient of a constant vector?	K1	CO4	2M
	h) Define the curl of a vector point function \vec{F} . When does, \vec{F} is said to be irrotational vector?	K1	CO4	2M
	i) Define work done by force \vec{F} . If \vec{F} is conservative, then what is the impact of it work done?	K2	CO5	2M
	j) State Gauss divergence theorem.	K1	CO5	2M
UNIT-I				
2.	a) Solve $\frac{dy}{dx} - \frac{2y}{x+1} = (x+1)^3$.	K3	CO1	5M
	b) If the temperature of the air is 30°C , and the substance cools from 100°C to 80°C in 10 minutes, find the temperature of the substance after 20 minutes.	K3	CO1	5M
OR				
3.	a) Solve $x \frac{dy}{dx} + y = x^3 y^6$.	K3	CO1	5M
	b) Determine the charge and current at any time in 't' in R-C circuit with $R = 100$ ohms, $C = 2$ farad and $E = 100$ volts given that $q(0) = 0$.	K3	CO1	5M

UNIT-II

4.	a)	Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$.	K3	CO2	5M
	b)	Solve $y^{11} + 4y^1 + 4y = 3 \sin x + 4 \cos x, y(0) = 1 \text{ and } y^1(0) = 0$.	K3	CO2	5M

OR

5.		In an L-C-R circuit, the charge q on a plate of a condenser is given by $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt$. The circuit is tuned to resonance so that $p = \frac{1}{\sqrt{LC}}$. If initially the current I and the charge q be zero, for small values of $\frac{R}{L}$, the current in the circuit at time t is given by $\frac{Et}{2L} \sin pt$.	K3	CO2	10M
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UNIT-III

6.	a)	Form a partial differential equation by the eliminating of arbitrary constants from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	K3	CO3	5M
	b)	Solve $x(y - z)p + y(z - x)q = z(x - y)$.	K3	CO3	5M

OR

7.	a)	Form the partial differential equation by eliminating the arbitrary functions from $z = f(2x + y) + g(3x - y)$	K3	CO3	5M
	b)	Solve $\frac{\partial^2 z}{\partial x^2} - 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = e^{2x-y} + e^{x+y} + \cos(x + 2y)$.	K3	CO3	5M

UNIT-IV

8.	a)	Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P = (1, 2, 3)$ in the direction of the line PQ where $Q = (5, 0, 4)$.	K3	CO4	5M
	b)	For a solenoid vector \vec{F} prove that $\text{curl curl curl curl } \vec{F} = \nabla^4 \vec{F}$.	K3	CO4	5M

OR

9.		If $\vec{r} = xi + yj + zk$ prove that $\text{div}(r^n \vec{r}) = (n+3)r^n$; $\text{curl}(r^n \vec{r}) = 0$. If \vec{r} is a solenoidal vector, find n?	K3	CO4	10M
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UNIT-V

10.	a)	If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along $x = t, y = t^2, z = t^3$.	K3	CO5	5M
	b)	Evaluate $\int_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 6z\vec{i} - 4\vec{j} + y\vec{k}$ and S is the portion of the plane $2x + 3y + 6z = 12$ in the first octant.	K3	CO5	5M

OR

11.		Verify Green's theorem for $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$.	K3	CO5	10M
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