

**PRAGATI ENGINEERING COLLEGE: SURAMPALEM
(AUTONOMOUS)**

I B.Tech I Semester Supplementary Examinations, July-2024

LINEAR ALGEBRA AND CALCULUS

(Common to all branches)

Time: 3 hours

Max. Marks: 70

Note:

- i. Question No. 1 shall contain 10 compulsory short answer questions (2 questions from each unit) for a total of 20 marks such that each question carries 2 marks.
- ii. In each of the questions from 2 to the last question, there shall be either/or type questions of 10 marks each.

Q. No.	Questions	BTL	CO	Marks
1.	a) Explain trivial and non-trivial solution for homogeneous system $AX = 0$.	K2	CO1	2M
	b) For what values of λ and μ the following system $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & \lambda - 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ \mu - 10 \end{bmatrix}$ has infinitely many number of solutions.	K2	CO1	2M
	c) Find the quadratic form related to symmetric matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & -5 & -1/2 \\ 1 & -1/2 & 2 \end{bmatrix}$.	K2	CO2	2M
	d) Find the characteristic equation for $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.	K2	CO2	2M
	e) Discuss the geometrical interpretation of Lagrange's mean value theorem.	K2	CO3	2M
	f) Write the Taylor's series of $\cos x$ about $x = \frac{\pi}{4}$.	K2	CO3	2M
	g) If $u = \log(x + y + z)$ where $x = e^t, y = \sin t, z = \cos t$, find $\frac{du}{dt}$.	K2	CO4	2M
	h) If $u = f(x + ay) + g(x - ay)$, then find $\frac{\partial^2 u}{\partial y^2}$.	K2	CO4	2M
	i) Evaluate $\int_{-1}^2 \int_{x^2}^{x+2} dy dx$.	K2	CO5	2M
	j) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^a \sin \theta \int_0^{\frac{a^2-r^2}{a}} r dz dr d\theta$.	K2	CO5	2M
UNIT-I				
2.	a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$ by reducing into normal form.	K3	CO1	5M
	b) Test the consistency of the homogeneous system of linear equations $x + 2y + 3z = 0; 3x + 4y + 4z = 0; 7x + 10y + 12z = 0$.	K3	CO1	5M
OR				
3.	a) Solve the system of equations by using Jacobi method $20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25$.	K3	CO1	5M

	b)	Verify Cauchy-Binet formula for $A = \begin{bmatrix} 1 & 3 & 5 & 6 \\ -3 & 5 & 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 7 & 8 \\ -9 & 2 \end{bmatrix}$.	K3	CO1	5M
UNIT-II					
4.	a)	Applying Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1} and A^4 .	K3	CO2	7M
	b)	If λ is an eigen value of a non-singular matrix A then show that λ^{-1} is an eigen value of a matrix A^{-1} .	K3	CO2	3M
OR					
5.		Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ into canonical form by orthogonal transformation. Hence find its rank, index, signature and nature.	K4	CO2	10M
UNIT-III					
6.	a)	Apply Lagrange's mean value theorem, to prove $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}, 0 < a < b < 1$.	K3	CO3	5M
	b)	Verify Cauchy's mean value theorem for the function $f(x) = \sin x, g(x) = \cos x$ in $\left[0, \frac{\pi}{2}\right]$.	K3	CO3	5M
OR					
7.	a)	Identify the applicability of Rolle's theorem for $\log \left[\frac{x^2+ab}{x(a+b)} \right]$ in $[a, b], 0 < a < b$.	K3	CO3	5M
	b)	Using Maclaurin's series, expand $f(x) = \tan x$ upto the term containing x^5 .	K3	CO3	5M
UNIT-IV					
8.	a)	Identify whether $u = 2x - y + 3z, v = 2x - y - z, w = 2x - y + z$ are functionally dependent or not, if so establish a relationship between them.	K3	CO4	5M
	b)	Apply the technique of Lagrange's undetermined multipliers establish a maximum value of $x^m y^n z^p$ subject to the condition $x + y + z = a$.	K3	CO4	5M
OR					
9.	a)	If $u = f(x - y, y - z, z - x)$ then utilize chain rule to prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	K3	CO4	5M
	b)	Expand x^y in powers of $(x - 1)$ and $(y - 1)$ upto terms of third degree.	K3	CO4	5M
UNIT-V					
10.		Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$.	K3	CO5	10M
OR					
11.	a)	Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} dz \, dx \, dy$.	K3	CO5	5M
	b)	Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} \, dx \, dy$ by changing variables in to polar coordinates.	K3	CO5	5M